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Open string decoupling and tachyon condensation

Gordon Chalmers

Argonne National Laboratory, High Energy Physics Division
9700 South Cass Avenue, Argonne, IL 60439-4815, USA
E-mail: chalmers@pcl9.hep.anl.gov

Abstract: The amplitudes in perturbative open string theory are examined as functions of the tachyon condensate parameter. The boundary state formalism demonstrates the decoupling of the open string modes at the non-perturbative minima of the tachyon potential via a degeneration of open world-sheets and identifies an independence of the coupling constants g_s and g_{YM} at general values of the tachyon condensate. The closed string sector is generated at the quantum level; it is also generated at the classical level perturbatively through the condensation of propagating open string modes on the D-brane degrees of freedom.

Keywords: Strong Coupling Expansion, Bosonic Strings, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Open string field theory contains closed strings in its quantum spectrum. It is conjectured that open string states decouple at a non-perturbative minima of the tachyon condensate [1] (further analyzed in [2, 3, 4]). Approximations of the tachyon potential in string field theory are found in,[1] [6]–[15], for example. Similar conjectures for the superstring are covered in [16, 17, 18]. In this letter, we examine scattering amplitudes in the mixed open/closed theory within the boundary state formalism developed in a number of works [19]–[23]; in this formalism we may explicitly demonstrate that scattering amplitudes containing open string modes decouple, or, equivalently, that Riemann surfaces with non-vanishing boundaries describing the worldsheet are exponentially suppressed with respect to the tachyon condensate; and we answer a question regarding how the closed string modes arise at the classical level. We also determine independence of the bosonic coupling constants in open string theory, g_s and $g_{\rm YM}$, and model them via bulk and boundary worldsheet expectation values. This independence of the string couplings generates broken conformal invariance that is found in off-shell open string field theory.

The open/closed string duality [24] generates closed strings in the quantum amplitudes of the open string theory (and also generates precise means to map non-abelian gauge theory amplitudes to gravitational amplitudes [25]). It is of interest to generate alternate means for finding closed string excitations including gravitation from a classical open string model. The role of the coupling constant relation $g_s = g_{\rm YM}^2$ in on-shell open string theory and the holomorphic/anti-holomorphic factorization of scattering amplitudes permits a direct comparison between unintegrated graviton amplitudes with gauge boson amplitudes. In the context of the BSFT

[1] Investigations of the tachyon potential in the context of dual models are found in [5]

(boundary string field theory) formalism, however, the coupling constant relation may be generalized: a mechanism for generating a closed string sector with non-interacting open strings is to take $g_{\rm YM} \to 0$. The boundary worldsheet interactions and the relation to the open string coupling constant breaks conformal invariance of the world-sheet theory, but may be related to a field redefinition in the target space-time. This limit in the target spacetime is akin to examining

$$\mathcal{L} = \int d^{26}x\sqrt{g}\left(\frac{1}{\kappa^2}R + \frac{1}{g_{\rm YM}^2}\,\mathrm{Tr}\,F^2\right), \tag{1.1}$$

the limit of fixed κ and varying $g_{\rm YM}^2$ at fixed α'. In the target spacetime theory, it is obvious that general values of the coupling constant generate consistent theories. This will be given a concrete description in terms of the open string theory in a non-trivial tachyon background in this letter. A nullified gauge coupling constant decouples open string modes.

Geometrically, there are two scalar deformations on the open string worldsheet that generate gauge coupling constants:

$$\int_{M_{g,h}} d^2z\sqrt{g}R\phi + \int_{\partial M_{g,h}} dz\, e\tilde{\phi}\,, \tag{1.2}$$

with R the bulk worldsheet curvature, the integral giving a topological invariant on genus g, $\int d^2z\sqrt{g}R = 2-2g-h$. The latter picks up a factor under non-area preserving diffeomorphisms, and the scaling may be absorbed in general under a scale change of the coupling $\tilde{\phi}$. Modulo the worldsheet renormalizations, these expectation values couple as closed and open string dilaton and the latter models the off-shell structure of perturbation theory in the open string theory via the boundary state field formalism. The path integral quantization is direct and may be performed explicitly in perturbation theory. The open string field theory with an off-shell tachyon is modeled by the critical bosonic theory with the above deformations. The introduction of the open string boundary term generates both off-shell open string theory in addition to the equivalent interpretation in which $g_s \neq g_{\rm YM}^2$. The latter may be interpreted as a breaking of the conformal invariance of the string away from the point in which $g_{\rm YM} = 0$ ($\tilde{\phi} = -\infty$) and $g_s \neq 0$ in which the closed sector remains as the consistent closed bosonic theory. This holds in perturbation theory order by order in the worldsheet expansion.

This work is organized as follows. In section 2 we examine the scattering amplitudes in the BSFT formalism and probe them as a function of the vacuum values ϕ and $\tilde{\phi}$. In section 3, we demonstrate that closed strings arise at the classical level in the non-perturbative regime of the tachyon vacuum. In section 4 we conclude with discussion.

2. Amplitudes in the BSFT of the string field

The worldsheet bosonic action of the bosonic string theory is described by

$$S_0 = \int_M d^2z\sqrt{g}g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} + \int_M d^2z\sqrt{g}R\phi\,. \tag{2.1}$$

The deformations that breaks worldsheet conformal invariance,

$$S[u_j, a] = \int_{\partial M} dze\left(-a + \sum_{j=1}^{26} u_\mu X_\mu^2\right), \tag{2.2}$$

describe the solitonic configuration [26]. The boundary conditions following from the above $S + S[u_j, a]$ are

$$n^\alpha \partial_\alpha X_\mu + u_\mu X_\mu = 0\,. \tag{2.3}$$

On a surface with boundary the former term arises from the boundary term,

$$\int d^2z\sqrt{g}\partial^\alpha\left(g^{\alpha\beta}\partial_\beta\chi\right) = \int dze\, a\,, \tag{2.4}$$

and represents the total derivative and boundary fluctuations on the open string worldsheet.

The tachyon potential in the open bosonic string theory has the approximate form [27]

$$V(T) = \exp\,(-a)(1+a)\,. \tag{2.5}$$

A target spacetime redefinition, $\exp(-a) = \phi^2$, changes the potential into

$$V(\phi) = \phi^2(1 - 2\ln\phi)\,. \tag{2.6}$$

Within the boundary state formalism, the tachyon condenses at the point $a \to \infty$ and $u_j \to 0$ (destabilizing the solitonic configuration), modeled by exponentiating zero momentum insertions.

The non-vanishing values of a and u_j break the conformal invariance on the boundaries of the Riemann surfaces. Greens functions are independent of the parameter a. However, as the variation of the boundary cosmological constant term is independent of X_μ. The conformal mappings change the functional form of the Greens functions as a function of u_j. On the disc with flat metric $ds^2 = dz^2 + d\bar{z}^2$ the two-point function with boundary conditions is

$$\begin{aligned} G(z_i, z_j; u) = &-\ln|z_i - z_j|^2 - \ln|z_i\bar{z}_j + 1|^2 + \frac{2}{u} - \\ &- 2u\sum_{k=1}^{\infty}\frac{1}{k(k+u)}\left[(z_i\bar{z}_j)^k + (\bar{z}_i z_j)^k\right] \end{aligned} \tag{2.7}$$

and on the stereographically projected sphere the Greens function, following from a coordinate transformation

$$z \longrightarrow \frac{1-z}{1+z} \tag{2.8}$$

takes on the form,

$$\begin{aligned} G(z_i, z_j; u) = &- \ln|z_i - z_j|^2 - \ln|z_i + z_j|^2 + \frac{2}{u} - \\ &- 2u \sum_{k=1}^{\infty} \frac{1}{k(k+u)} \left[\left(\frac{[1-z][1-\bar{w}]}{[1+z][1+\bar{w}]} \right)^k + \left(\frac{[1-\bar{z}][1-w]}{[1+\bar{z}][1+w]} \right)^k \right]. \end{aligned} \tag{2.9}$$

The limit in which $u_j = 0$ generates conformally invariant Greens functions (after subtracting the zero mode) as in the critical string. The limit of these Greens functions on the boundary is

$$G(\theta_1, \theta_2; u) = 2 \sum_{k \in \mathbb{Z}} \frac{1}{|k| + u} e^{i(\theta_1 - \theta_2)}, \tag{2.10}$$

together with the first derivative,

$$\partial G(\theta_1, \theta_2; u) = 2i \sum_{k \in \mathbb{Z}} \left[\frac{k}{|k| + u} \right] e^{i(\theta_1 - \theta_2)}. \tag{2.11}$$

The OPE associated with the two-point function is straightforward to carry out and generates an identical structure to the undeformed case. Similar Greens functions on multi-genus Riemann surfaces are obtained from the prime form $E(z_i, z_j)$. To explore the tachyon condensation limit, however, we require $u_j = 0$ and the multi-genus Greens functions are conformally invariant in this limit explicitly (they do not depend on the parameter a).

The scattering amplitudes are,

$$A(k_j, \epsilon_j) = \int \frac{[dg][dX]}{N} e^{-(S_0 + S[u_j, a])/\alpha'} \prod_{m=1}^{n} V_{k_m}, \tag{2.12}$$

modulo a renormalization of the boundary coupling compensating for the conformal invariance being broken at the boundary. The local OPE generates poles identical to the free theory, and the mass levels do not shift as a function of u_j: they are trivially independent of the deformation parameter a, because the Greens functions are independent of this parameter. The term $S[0, a]$ appears to break conformal invariance; the choice of the gauge fixing of the diffeomorphisms generates different exponential factors associated with this term in the world-sheet theory. In the target spacetime this ambiguity from

$$\exp\left(-a \int dz\, e + S_0 \right), \tag{2.13}$$

is related to the ambiguity in defining an off-shell extension of string theory. However, for any choice of metric on the world-sheet a field redefinition of a may be performed that removes the ambiguity. In the target space-time theory this is a relabeling of coordinates that parameterizes the moduli space of tachyon vacua. In the target space-time theory, the interaction appears conformal invariant.

Thus the interaction generates on a worldsheet,

$$\int dg \exp\left(-a\int dz\, e\right) = \exp\left(-a_{\hat{g}}(1+h)\right), \tag{2.14}$$

where $\hat{g}$ denotes the conformal class, and $a_{\hat{g}}$ is a redefined tachyon expectation value.

As an example, consider the four-point function $A_4(k_i, \epsilon_i)$ evaluated on the sphere with non-vanishing u_j parameters. We take the limit in which $u_j = 0$; the conformal breaking only enters into the conformal prefactor associated with the open string coupling. The Koba-Nielson representation of the amplitude is

$$A = e^{-al}\int \prod_{i,j=1}^{4} \exp\left(-\alpha k_i\cdot k_j G_{ij} + \epsilon_{[i}\cdot k_{j]}\dot{G}_{ij} + \epsilon_i\cdot\epsilon_j \ddot{G}_{ij}\right)|_{\text{multi-linear}} \tag{2.15}$$

and as an expansion in the polarizations it generates the tree-level amplitude process. The cosmological constant term in the exponential from $S[0,a]$, al, generates the exponential suppression at the boundary and thus the OPE is independent of a.

The gauge fixed n-point tachyon amplitude is similarly,

$$\begin{aligned} A_4 = {} & e^{-a\beta}(z_{n-2}-z_{n-1})(z_{n-1}-z_n)(z_{n-1}-z_n)\times \\ & \times\int\prod dz_j \prod_{i\neq j}\exp\left(-G_{ij}s_{ij}\right)|_{z_1=0,z_2=1,z_n=\infty} \end{aligned} \tag{2.16}$$

with $s_{ij} = (k_i + k_j)^2$. Mixed scattering between the gauge bosons and tachyon modes are found by replacing the jth polarization vector in the above with zero and substituting in the mass shell condition of the tachyon, $k_j^2 = m^2$.

The expansion of the perturbative series in the genus expansion is

$$A_n(k_i, \epsilon_i; a) = \sum e^{-p\phi}e^{-a_{\hat{g}}}A_n^{(m)}(\epsilon, k_i), \tag{2.17}$$

with ϕ the closed dilatonic factor, and $A_n^{(m)}$ the n-point genus m sub-amplitude. Consistency with unitarity requires the relative normalization between the open couplings of $e^{-a_{\hat{g}}}$ at different orders in the expansion.

Next, we comment on the behavior of the poles and residues as a function of the tachyon vacuum at non-vanishing values of u. The Greens function in (2.9) has the expansion at small values of u

$$\begin{aligned} e^{\alpha' G(z_1,z_2)} = {} & |z_1-z_2|^{2\alpha'}|z_1+\bar{z}_2|^{2\alpha'}e^{2/u}\times \\ & \times\sum_{m=0}^{\infty}(-2u)^m\left[\sum_{k=1}^{\infty}\frac{1}{k(k+u)}\left(\frac{(1-z)(1-\bar{w})}{(1+z)(1+\bar{w})}\right)^k + \right. \\ & \left. + \left(\frac{(1-\bar{z})(1-w)}{(1+\bar{z})(1+w)}\right)^k\right]^m \end{aligned} \tag{2.18}$$

and the poles do not shift as a function of u. However, the residues of the poles have a non-trivial u structure, and the expansion of a pole changes as

$$\sum_{k=1}^{\infty} \frac{\alpha(k,u;s)}{s-m_k}\,, \tag{2.19}$$

with $\alpha(k,u;s) = b_n + \alpha n + g(u) + nh(u)$. This signals that the tachyon medium has a dielectric effect on the angular momentum associated with the pole.

3. Closed strings via confinement on the brane

Non-perturbative mechanisms have been analyzed inducing confinement of the open strings, see for example [4, 28]. The presence of the Dirichelet boundary conditions reflect a soliton in the spacetime, and we present two microcopic mechanisms of the dynamics of the open strings into closed counterparts. The first mechanism reflects the Hilbert space. The second represents the forces acting on the endpoints of the open strings and not the closed strings themselves; however, the force in the latter mechanism does indicate a confinement into closed strings.

One mechanism for the generation of closed strings at the classical level is found through the limit $a \to \infty$. The open strings propagating on the soliton parameterized by the Dirichelet boundary conditions span a worldsheet with boundary, and as such must degenerate in the limit of $a \to \infty$ (since the coupling is $e^{-al_{\rm bdy}}$).[2] This limit provides a dynamical means for the doubling of the Hilbert space at the classical level.

Furthermore, at non-vanishing values of u, the force between two points on the soliton may be computed from the tree-level Greens functions $G(z_1, z_2; a, u)$. At values $u \to 0$, the Greens function creating the force between two test particles on the brane is

$$G(z_1, z_2; u) = -\ln|z_1 - z_2|^2 - \ln|z_1\bar{z}_2 + 1|^2 + \frac{2}{u} + \mathcal{O}(u)\,. \tag{3.1}$$

As $u \to 0$ a potential from the zero mode binds the two points on the soliton, closing the open string degrees of freedom on the soliton as $u \to 0$ and as $a \to \infty$. This confining mechanism (linear in $1/u$) dynamically generates the closed string degrees of freedom at the classical level, as the tachyon condenses and the brane evaporates.

4. Conclusions

The perturbative amplitudes in the boundary state formalism are examined together with the pole structure of the amplitudes and the functional dependence of a and u_j. In the limit in which $a \to \infty$, the amplitudes associated with bounded Riemann

[2] This is also examined in the context of "hole cutting operators" in [28]

surfaces (i.e. open string worldsheets) are exponentially suppressed as a function of the tachyon expectation value. In the limit of the tachyon mimima, only the closed string sector remains, in agreement with the Sen conjecture. The mechanism for the decoupling of the open strings is perturbative in the boundary state formalism, and generalizes to off-shell the open-closed duality. An interpretation of the parameter a is a decoupling of the two couplings g_s and $g^2_{\rm YM}$ in the string, which leads to a breaking of conformality on the world-sheet. The ambiguity associated with the world-sheet non-conformal invariant term, $-a\int dze$, along the boundaries of the Riemann surfaces may be absorbed by a field redefinition of the tachyon expectation value labeling the vacua. This procedure holds order by order in perturbation theory and multi-genus results may be performed in a similar fashion.

The tachyon condensation in this framework indicates that the closed string excitations are perturbative. The closed string modes arise as bound states of the open string states propagating on the brane as the tachyon condenses. The decoupling of the open worldsheet surfaces generates a dynamical means for doubling the Hilbert space, as is required for consistency with quantum open/closed duality.

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